

# Procedure for Determination of LTTR for Thin Specimens of Cellular Plastic Thermal Insulations

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## Introduction

This procedure is intended for cellular plastic insulations that are too thin to undergo evaluation of LTTR by slicing and scaling. The thicknesses of interest are less than 15 mm. The procedure requires thermal measurements for full-thickness specimens over time (t). A sequence of measurements provides the thermal resistance, R, as the age of the specimen increases. The measured R-values are used to calculate the ratio  $R/R_0$  where  $R_0$  is the initial thermal resistance of the specimen (at  $t=0$ ).

The set of test data  $\{t, R/R_0\}$  can be described by Equation 1 with the parameters  $A$  and  $B$  determined in accordance with the Method of Least Squares. Time,  $t$ , is entered in days.

$$\frac{R}{R_0} = 1 - A \cdot (1 - e^{-B \cdot t}) \quad (1)$$

Equation (1) approaches a constant value as  $t$  becomes large and has the correct value at  $t=0$ .

$$\frac{R}{R_0} = 1 \text{ when } t = 0 \text{ and } \lim_{t \rightarrow \infty} R/R_0 = 1 - A \quad (2)$$

The determination of  $A$  and  $B$  involves the following definitions.

$$Y = 1 - \frac{R}{R_0} \quad (3)$$

$$X = (1 - e^{-B \cdot t}) \quad (4)$$

$$Y = A \cdot X \quad (5)$$

The determination of the parameters  $A$  and  $B$  is a two-step process: (1) choose a  $B$  value and calculate an  $X$  for each  $t$ . Determine the Least Square value for  $A$ . There is an  $A$  value that corresponds to each  $B$  that is selected. (2) Determine the  $B$  value that minimizes the summation:

$$W = \sum_{i=1}^5 (Y_i - A \cdot X_i)^2 \quad (6)$$

A value for the summation in (6) can be calculated for each  $B$  and the corresponding  $A$ . The values for  $W$  can be described by a quadratic in  $B$  with the minimum determined from the  $B$  that yields  $dW/dB=0$ . This process provides parameters  $A$  and  $B$ . The calculation of  $A$  and  $B$  from a five-point data set (plus a value at time 0) can be performed using an EXCEL program that is provided with this procedure. Equation (1) with specific values for  $A$  and  $B$  is used to calculate  $R/R_0$  at time  $t=5$  years

## Summary of the Test Procedure

First, the thermal resistance of the full thickness product with facing intact is measured periodically for a period up to 700 days (ca. 2 years); the measuring times are 10-14 days after manufacturing, and then after 100, 250, 400, 550 and 700 days (+5 days in each case). For product thinner than 12mm, stack 2 specimens for all thermal resistance measurements. Determine the thermal resistance of four specimens in accordance with test method ASTM C518, Standard Test Method for Steady-State Thermal Transmission Properties by Means of the Heat Flow Meter Apparatus, or ASTM C177, Standard Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded-Hot-Plate Apparatus, using  $24 \pm 2$  °C mean temperature and a temperature difference of  $22 \pm 2$  °C. Second, the data are entered into the Excel spreadsheet to calculate the 5-year thermal resistance value as the LTTR value.

## How to Use the Excel Spreadsheet.

There are two tabs. The tab Input and Output is the tab to where the entry of data is to be done. The data input (in red cells) procedure for the EXCEL program is as follows.

- 1) At each measuring time  $t$  (in days), divide the measured R-Value ( $R$ ) by the R-Value measured at day 10-14 ( $R_0$ ) to calculate  $R/R_0$  ratio.
- 2) Enter data for  $t$  and  $R/R_0$  in cells  
B6 - C6            B cells are for time in days  
B7 - C7            C cells are for  $R/R_0$   
B8 - C8  
B9 - C9  
B10 - C10
- 3) Enter  $R_0$  in cell C3
- 4) The figure in lines 6 to 19 is important. This is the part of the program where the parameter  $B$  is determined. If the minimum in the curve does not appear in the figure, then the range of  $B$  values can be easily shifted. The smallest  $B$  can be entered in cell F3 (Min  $B$ ) and the delta  $B$  in cell H3 (Del  $B$ ). The program will immediately reassign  $B$  values and repeat the calculation.
- 5) There is a step in the calculation that requires transfer of numbers from a figure to specific cells. The figure in lines 6 to 19 contains an equation that relates a goodness-of-fit quantity to the parameter  $B$ . The program will select the minimum in the  $B$  curve.  
Cell D21: transfer the coefficient of  $x^0$  (the constant term) from the equation in the figure  
Cell D22: transfer the coefficient of  $x$  from the equation in the figure  
Cell D23: transfer the coefficient of  $x^2$  from the equation in the figure  
The final values for  $A$  and  $B$  that allows calculating LTTR result do not show up in cells H21 and H22 until you complete Step 5.
- 6) The calculated values for  $R/R_0$  compared to experimental values appear in lines 26-32.
- 7) The LTTR result appears in cell E37

- 8) A “goodness-of-fit” parameter  $R^2$  appear in cell M26.  $R^2$  near 1.000 is a desired output. In most cases, the  $R^2$  can be improved by reducing the del B in cell H3. This could require a shift in B in order to keep the minimum in the curve in the figure.

The result for the analysis appears in cell E37 as the LTTR value. There are two figures that show the experimental data compared to the curve fit; first, in lines 29 to 42 that is showing a comparison of experimental and calculated values for  $R/R_0$  and; and second, in lines 44 to 57 that is showing the difference between experimental and calculated as % of experimental.

### Example

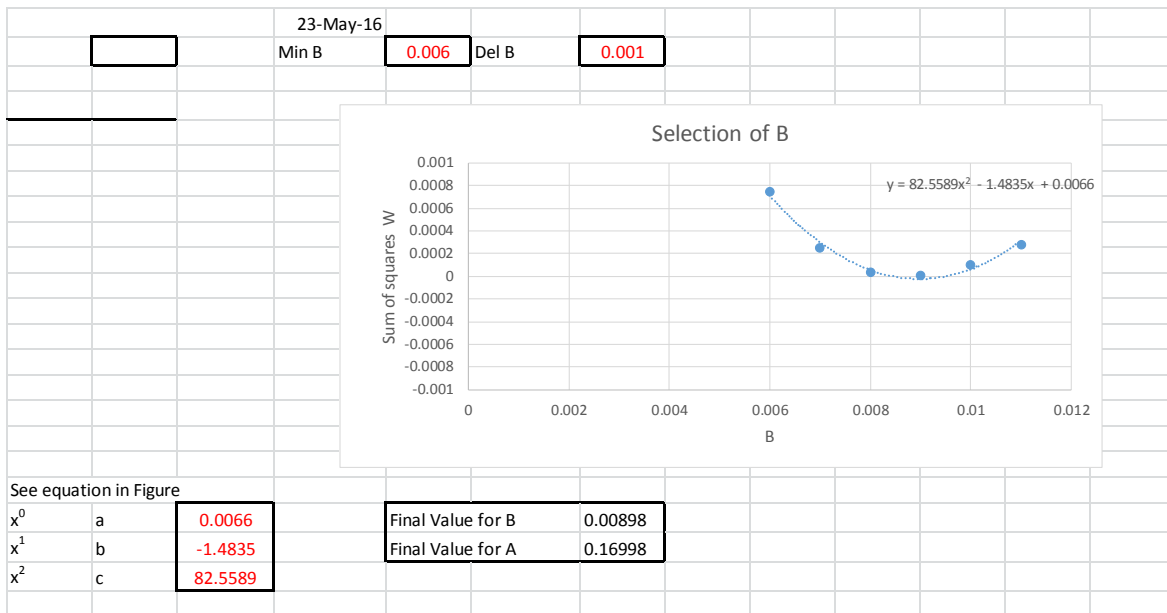
The following is offered as an example. The data are not related to any particular set of measurements. The input of measured values for  $R/R_0$  in cells B6 – C10 is shown in Figure 1.  $R_0$  is set at 2.5

Figure 1. Section of Program showing Data Input

t (days)	$R/R_0$
95	0.904
125	0.887
250	0.848
500	0.831
800	0.829

The program is set for B selection from 0.006 to 0.011. This can be changed by increasing or decreasing the number in cells F3 (Min B) and/or H3 (Del B). The figure in line 6 to 19 (Selection of B) shows how W (the sum of squared deviations) varies as B is increased. The coefficients in the equation in the figure are manually transferred to cells D21 ( $x^0$ ), D22 ( $x^1$ ) and D23 ( $x^2$ ). The optimum values for A and B are identified as Final values.

Figure 2. Section of Program Showing the Selection of Parameter B



The LTTR value is shown in Cell E37

Figure 3. Calculated Results for R/R0

Years (t)	Days	Result	
5	1826.25	0.830	R*or R/R <sub>0</sub>
		2.075	LTTR

The agreement between the experimental R/R<sub>0</sub> and values calculated using Equation (1) with the final values for A and B are shown in Figure 4. The percent difference between the calculated and experimental values are displayed in Figure 5.

Figure 4. Comparison of Experimental and Calculated Values for R/R<sub>0</sub>

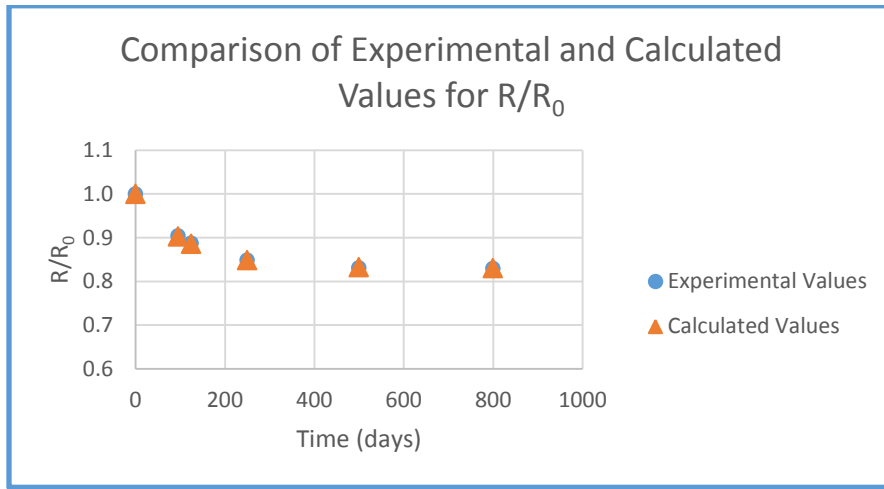
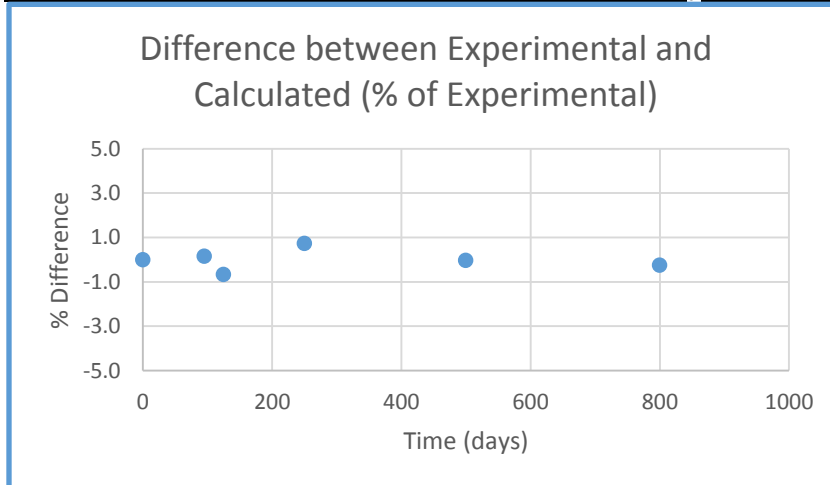


Figure 5. Differences Between Experimental and Calculated R/R<sub>0</sub> as % of Experimental



A “goodness-of-fit” parameter R<sup>2</sup> appear in Cell M26 (see Figure 6).

Figure 6. Section of the program showing R<sup>2</sup>

t	R/R <sub>0_exp</sub>	R/R <sub>0_calc</sub>	diff	%diff	S(y <sub>i</sub> - y <sub>calc</sub> ) <sup>2</sup>	S(y <sub>i</sub> - y <sub>ave</sub> ) <sup>2</sup>	R <sup>2</sup>
95	0.904	0.90241	-0.002	-0.17549	2.52E-06	0.001816	0.99969
125	0.887	0.88531	-0.002	-0.19072	2.86E-06	0.000651	
250	0.848	0.84800	0.000	0.00014	1.4E-12	0.000139	
500	0.831	0.83192	0.001	0.11052	8.43E-07	0.000777	
800	0.829	0.83014	0.001	0.13798	1.31E-06	0.000879	
Added	0	1	1.000	0.0000	0	0.019656	

A “real” set of measurements will likely shown larger deviations that the preceding example since experimental scatter will be included in the results.

A copy of the EXCEL program is a companion document to this discussion.

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